This bibliography is not intended to be complete but rather gives the main literature and web resources about biplots so that the reader can continue to learn about this method.

The term “biplot” originates in Ruben Gabriel’s *Biometrika* paper in 1971:


This paper, which at the time of writing has 1008 citations on Google Scholar and 682 on the Science Citation Index (ISI Web of Knowledge), is widely regarded as the origin of the idea. It is worthwhile to repeat its abstract:

“Any matrix of rank two can be displayed as a biplot which consists of a vector for each row and a vector for each column, chosen so that any element of the matrix is exactly the inner product of the vectors corresponding to its row and its column. If a matrix is of higher rank, one may display it approximately by a biplot of a matrix of rank two which approximates the original matrix. The biplot provides a useful tool of data analysis and allows the visual appraisal of the structure of large data matrices. It is especially revealing in principal component analysis, where the biplot can show inter-unit distances and indicate clustering of units as well as display variances and correlations of the variables.”

A less cited paper by Ruben Gabriel, but nevertheless one of my favourite ones on the biplot, appeared the following year in the *Journal of Applied Meteorology* (Ruben was also well-known for his work as a statistician in weather modification projects):


In this paper he gives the biplot associated with linear discriminant analysis, also known as canonical variate analysis. He also talks about the vectors linking pairs of variables in a biplot (like the “links” in log-ratio analysis).
Another gem is by Dan Bradu and Ruben Gabriel in *Technometrics* in 1978:


In this paper they show how certain models lead to points lying in straight lines in the full space of the data, and thus approximately in a biplot that has a good fit to the data. Thus a subset of row points and/or column points lying in a straight line in a biplot suggest models in that submatrix of the data. In addition, orthogonality of the lines suggests a simpler model.

All the above papers are required reading for those interested in the origins of the technique.

Other authors also had the idea of adding variables to an existing configuration of points to make joint displays, although they did not call them biplots. For example, Doug Carroll’s vector model for preferences is a biplot:


Only one book exists to date specifically on the topic of biplots, by John Gower and David Hand:


This book is very complete, both on linear and nonlinear biplots, giving a rigorous theoretical treatment of the subject. Another book by John Gower is with co-authors Suguett Gardner-Lubbe and Niel le Roux:


As far as the vast literature on the singular value decomposition (SVD) is concerned, I mention only two sources, by the author of one of the landmark algorithms for the SVD, Gene Golub in 1971, which seems to be an important year for the biplot:

and the other a classic book by Paul Green and Doug Carroll, originally published in 1976, which was the first time I saw the geometric interpretation of the SVD (called “basic structure” by these authors)—this book is invaluable as a practical introduction to matrix and vector geometry in multivariate analysis:


Most books or articles that treat the methods presented in this book will have a section or chapter on biplots and their interpretation in the context of that method. This is just a tiny selection of some of the literature that can be consulted, and by no means the primary references:

**Principal component analysis**

**Log-ratio analysis (unweighted form)**

**Log-ratio analysis (weighted form)**

**Correspondence analysis**

**Multiple correspondence analysis**

**Discriminant analysis/centroid biplots**
Constrained biplots


Finally we give some resources on the internet, on R packages relevant to this book (in alphabetic order of package names).


And some relevant websites:

http://www.multivariatestatistics.org

Supporting website for the series of statistics books published by the BBVA Foundation, including the present book *Biplots in Practice*, with glossary of terms and chapter summaries in Spanish, as well as supplementary material such as animated graphics and links to the data sets and R code.
Correspondence Analysis and Related Methods Network, with R scripts and data from this book, from *Correspondence Analysis in Practice*, Second Edition, and from *Multiple Correspondence Analysis and Related Methods*.

Jan de Leeuw’s website for the Gifi system (centred around multiple correspondence analysis and related methods) and R functions

Michael Crawley’s material from his book *Statistics: an Introduction Using R*

Source of many data sets from the International Social Survey Program

The R project for statistical computing

Jari Oksanen’s website for the *vegan* package in R, a very complete package which includes PCA, CA, CCA and many more multivariate methods, as well as permutation tests.

Website with data sets from book *Modern Multidimensional Scaling* by Ingwer Borg and Patrick Groenen

Website of José Luis Vicente Villardon’s biplot software for biplots and simple correspondence analysis